Expected returns, risk premia, and volatility surfaces implicit in option market prices

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Extension of current literature

A pure exchange economy extending Rubinstein (1976) illustrating how the jump-diffusion option pricing model of Merton (1976) is altered when jumps are correlated with diffusive risks.

- Covariance of diffusive pricing kernel with jumps of the pricing kernel, $\sigma_{cy_c}\sqrt{T}$
- Covariance of diffusive pricing kernel with price jumps, $\sigma_{cy}\sqrt{T}$

Extension of current literature cont.

- Covariance of diffusive price with jumps of pricing kernel, $\sigma_{sy_c}\sqrt{T}$
- Covariance of diffusive price with price jumps, $\sigma_{sy}\sqrt{T}$

Time T Consumption and Price

$$
\begin{aligned}
C_T &= exp\left(ln(C_0) + \mu_c T - \frac{\sigma_c^2}{2} T + \sigma_c B_c(T) + \sum_{i=1}^{N(T)} Y_{c,i} \right) , \\
S_T &= exp\left(ln(S_0) + \mu T - \frac{\sigma^2}{2} T + \sigma B(T) + \sum_{i=1}^{N(T)} Y_i \right)
\end{aligned}
$$

Mean and covariance structure

$$\begin{bmatrix} \sigma_c B_c(T) \\ \sigma B(T) \\ Y_{c,i} \\ Y_i \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ \alpha_c \\ \alpha \end{bmatrix}, \begin{bmatrix} \sigma_c^2 T & \sigma_{cs} T & \sigma_{cy_c}\sqrt{T} & \sigma_{cy}\sqrt{T} \\ & \sigma^2 T & \sigma_{sy_c}\sqrt{T} & \sigma_{sy}\sqrt{T} \\ & & \gamma_c^2 & v_{cs} \\ & & & \gamma^2 \end{bmatrix} \right)$$

Euler Equation

$$P_0 = E\left[\frac{U'(C_T)}{U'(C_0)}\phi(S_T) \mid \mathcal{F}_0\right]$$

The equilibrium price of an arbitrary asset with payoff $\phi(S_T)$ at time T.

Power Utility

$$U_t(C_t) = \rho^t C_t^{1-b}/(1-b)$$

Power utility function of the representative agent

- ρ is the time discount factor
- b is the coefficient of proportional risk aversion.

Pricing Kernel

$$\psi(C) = \frac{U'(C_T)}{U'(C_0)} = \rho^T \left(\frac{C_T}{C_0}\right)^{-b}$$

Representative agent's marginal rate of substitution C_T for C_0 with power utility

Pricing Kernel

$$\psi(C) = exp\left(ln(\rho)T - b\mu_c T + b\frac{\sigma_c^2}{2}T - b\sigma_c B_c(T) + \sum_{i=1}^{N(T)} (-bY_{c,i}) \right)$$

Pricing kernel given assumed dynamics and power utility

CCAPM

$$r = \mu - b\sigma_{cs} + \lambda \left[e^{-b\alpha_c + \frac{b^2}{2}\gamma_c^2 + b^2\sigma_{cy_c}\sqrt{T} + \alpha + \frac{\gamma^2}{2} + \sigma_{sy}\sqrt{T} - b\sigma_{sy_c}\sqrt{T} - b\sigma_{cy}\sqrt{T} - bv_s} \right.$$

$$\left. - \lambda \left[e^{-b\alpha_c + \frac{b^2}{2}\gamma_c^2 + b^2\sigma_{cy_c}\sqrt{T}} - 1 \right] \right.$$

Linear Approximations

$$E(R_S(T)) = rT + EJRP(T) + EDRP(T)$$

where:

$$\begin{array}{llcl} (1) & E(R_S(T)) & = & \mu T + \lambda T[e^{\alpha+0.5\gamma^2+\sigma_{sy}\sqrt{T}} - 1] \\ (2) & EJRP(T) & = & -\lambda T[e^{-bv_{sc}-b\sigma_{cy}\sqrt{T}} - 1] \\ (3) & EDRP(T) & = & b\sigma_{cs}T - \lambda T[e^{-b\sigma_{syc}\sqrt{T}} - 1] \end{array}$$

GJD Option pricing model

$$P_c = \sum_{n=0}^{\infty} \frac{(\lambda' T)^n e^{-\lambda' T}}{n!} \left(S_0 N(d_1(n)) - K e^{-r_n T} N(d_2(n)) \right)$$

GJD Continued

$$
\begin{aligned}
d_1(n) &= \frac{ln\left(\frac{S_0}{K}\right) + (r_n + \frac{\sigma_n^2}{2})T}{\sigma_n\sqrt{T}}, \qquad d_2(n) = d_1(n) - \sigma_n\sqrt{T}, \\
\lambda^{'} &= \lambda\beta_1, \\
\sigma_n^2 &= Max\left\{n\gamma^2/T + 2n\sigma_{sy}/\sqrt{T} + \sigma^2, 0\right\}, \\
r_n &= r + \lambda\left(\beta_2 - 1\right) - \lambda\left(\beta_1 - 1\right) + \frac{n}{T}ln\left(\frac{\beta_1}{\beta_2}\right), \\
\beta_1 &= e^{-b\alpha_c + \frac{b^2}{2}\gamma_c^2 + b^2\sigma_{cyc}\sqrt{T} + \alpha + \frac{\gamma^2}{2} + \sigma_{sy}\sqrt{T} - b\sigma_{syc}\sqrt{T} - b\sigma_{cy}\sqrt{T} - bv_{sc}}, \\
\beta_2 &= e^{-b\alpha_c + \frac{b^2}{2}\gamma_c^2 + b^2\sigma_{cyc}\sqrt{T}}.
\end{aligned}
$$

Methodology

- Non-parametric test of difference from zero of median GJD implied parameter values
- Best bid/offer quote mid-point reported by OptionMetrics.
- Select GJD implied parameter values that minimize sum-of-squared price errors for observation date.

Data

- Non-overlapping sample call & put S&P 500 index option quotes
- One-month option quotes observed one-month prior to expiration
- 6,430 option quotes from 124 observation dates January 1996 - April 2006
- Data screens; quote midpoint > $3/8, positive volume, open interest, no-arbitrage boundaries.

GJD Implied Parameter Values

Equilibrium pricing by substitution methodology yields GJD implied parameter values of actual/objective distributions

- Aggregate consumption process
- Mean and covariance structure of uncertainty in aggregate consumption and stock price
- Implied parameter values used to produce estimates of risk premia.

$\sigma_{sy_c}\sqrt{T} < 0$

Implied covariance between diffusive price level and price jumps

- Reduces $E(R_S(T))$
- $E(R_S(T))$ a nonlinear function of T
- Non-monotonic term structure of Black-Scholes implied volatilities.

Estimates of EJRP

- Pan (2002) 18.4%
- Eraker (2004) 6%
- Broadie, Chernov, Johannes (2007) 2-4%
- Santa Clara, Yan (2006) 6.9%

$b\sigma_{cy}\sqrt{T} > 0$

Jump risk premia due to covariance between diffusive price kernel and price jumps

- Increases EJRP 6.8% to 12.1%
- EJRP a nonlinear function of T
- Produces sneers or slope of the Black-Scholes implied volatility surface.

$b\sigma_{sy_c}\sqrt{T} > 0$

Covariance between diffusive return and jumps in pricing kernel

- Increases EDRP
- EDRP a nonlinear function of T.

Jump Intensity λ

$\hat{\lambda}$, $\hat{\mu}$, $\hat{\alpha}$ can be used to estimate time till market crash implied by GJD parameter values

- $d\ln(S) < -10\%$ $\frac{1}{3.03}$ years
- $d\ln(S) < -20\%$ $\frac{1}{24.76}$ years

Santa Clara and Yan (2006)

- $d\ln(S) < -9.8\%$ $\frac{1}{1.26}$ years

Preference parameter, b

Coefficient of proportional risk aversion

- Median GJD model implied b = 6.56
- Bliss and Panigirtzoglou (2002) 3.85